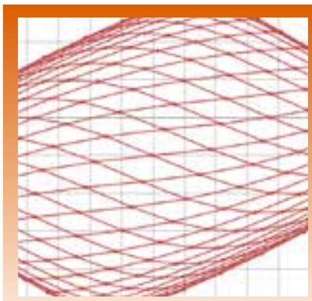

Slip Angle provides a summary of OptimumG's seminars

Getting to grips with your yaw moments

In the first instalment in a new series OptimumG vehicle dynamics engineer Claude Rouelle takes us through some yaw angle basics

One important part of a racecar performance engineer's job is lap time simulation. Simulating and comparing the effect of car design and set-up parameters on the lap time is essential. With the many inputs and outputs that exist in such simulation, it is always worth having metrics other than the lap time to know if and why we improve the car's performance. Metrics such as grip, balance, control and stability on entry and at the limit issued from the yaw moment versus lateral acceleration method, created by Bill Milliken in 1953, provide such criteria.

Let's start with a question here. What is the yaw velocity of your car on a skip pad (circular handling pad)? Many Formula Student participants in design judging and even several professional race engineers incorrectly answer this question. Most of the time their answer is 'zero'. Wrong. That is because they mix the definition of the yaw velocity and the speed of the CG slip angle ß (the speed at which, in top view, the angle of the car's longitudinal axis changes with the tangent of the trajectory). On a skid pad we can assume that the yaw angle (some could call it the car attitude angle) is constant and therefore the yaw angle speed is zero. But despite very similar words used in their description, 'yaw angle speed' and 'yaw velocity' are not the same entities. To help students rethink their answer about skid pad yaw velocity, we can ask them to simply wonder if 360 degrees (one skip pad lap) divided by their car lap time could be the right answer …

Let's go back to the basics. We know that **$A = V^2/R$** (1), A being the car lateral acceleration (in m/sec^2), V the car speed (in m/sec) and R the radius of the skip pad (in m). We also know that **$V = rR$** (2), r being the yaw velocity (in rad/sec). If we put equations (1) and (2) together we get that **$r = A/V$** (3). Equation 3 is in fact incomplete. A more accurate definition of the yaw velocity is **$r = A/V + dß/dt$** (4), ß being the chassis slip angle (in rad). dß/dt, will be given by the derivative of the signal of the slip angle sensor.

On a skip pad we can assume we are in steady state condition and that V, A, and R are constant, that the chassis slip angle ß is constant too and $dß/dt = 0$.

Therefore, the yaw velocity r is constant too. If the yaw velocity is constant, the yaw acceleration must be zero.

Figure 1a shows a simplified mass point car on a skid pad. **Figure 1b** shows a car with a constant slip angle ß. **Figure 1c** shows the same car on the same

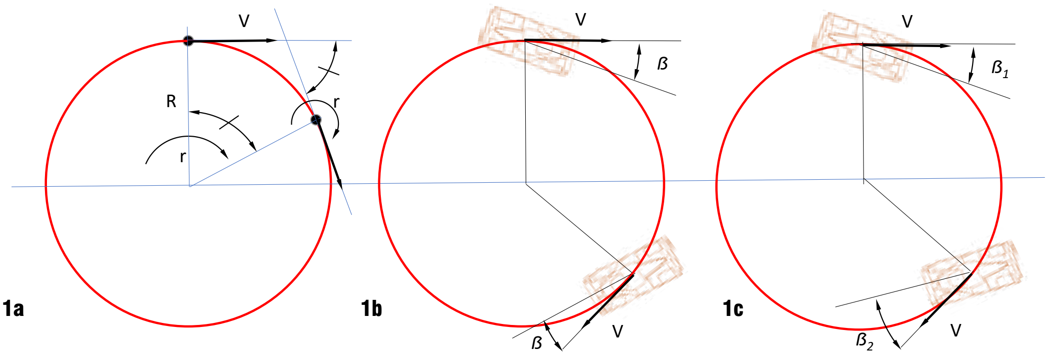

Figure 1a: Skip pad. Steady state mass point. Constant speed V, radius R and yaw velocity. The yaw velocity is nothing else than 360 degrees divided by the lap time
Figure 1b: Skip pad. Steady state vehicle. Same as 1a: constant yaw velocity r and constant CG slip angle ß. This is showing a racecar exhibiting a constant slip angle
Figure 1c: Skip pad. Transient vehicle. A, V and R are constant, CG slip angle varies

Here OptimumG has installed one of its slip angle sensors on the back of a GT car

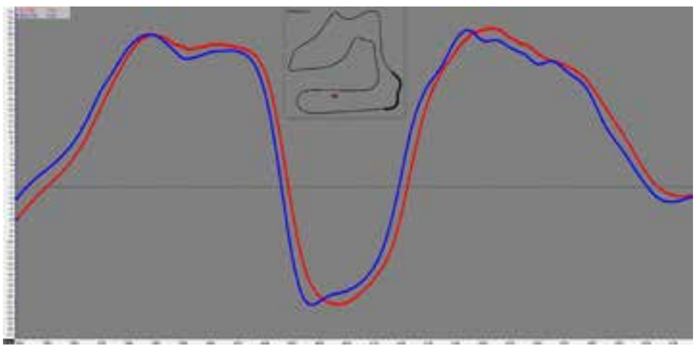

Figure 2: The difference between A/V match channel (the red trace) and the gyro signal (blue trace) is the slip angle speed dß/dt

Yaw angle speed and yaw velocity are not the same entities

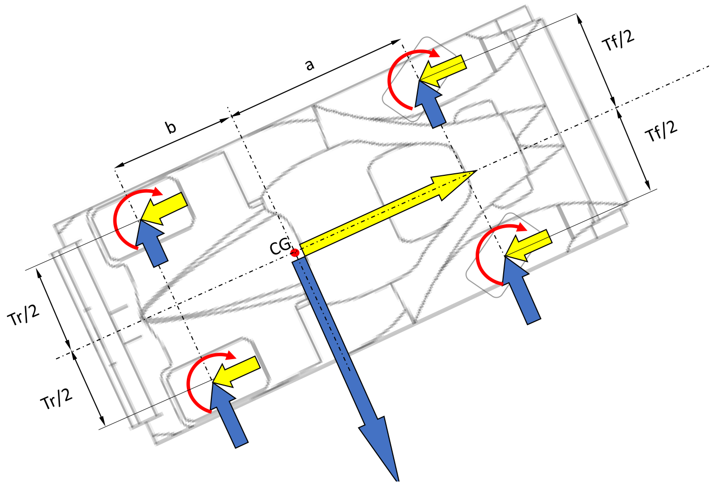

Figure 3: The 12 causes of yaw moment: four tyre lateral forces Fy, four tyre longitudinal forces Fx, and four tyre self-alignment torques Mz

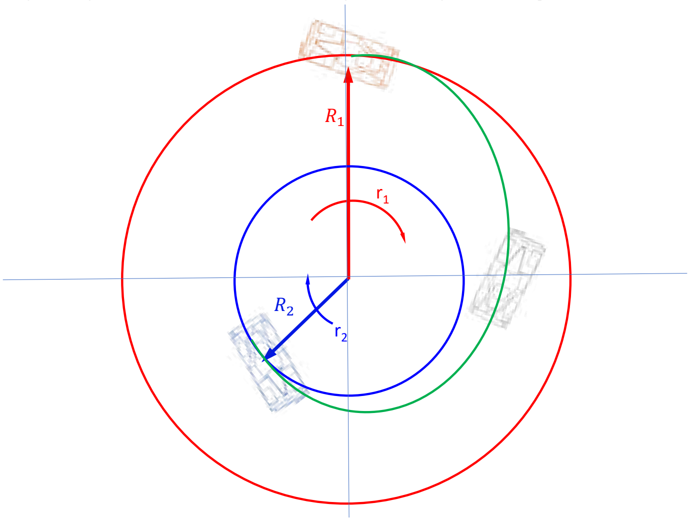

Figure 4: Going from one pad to another requires yaw moment N = Izz (dr/dt)

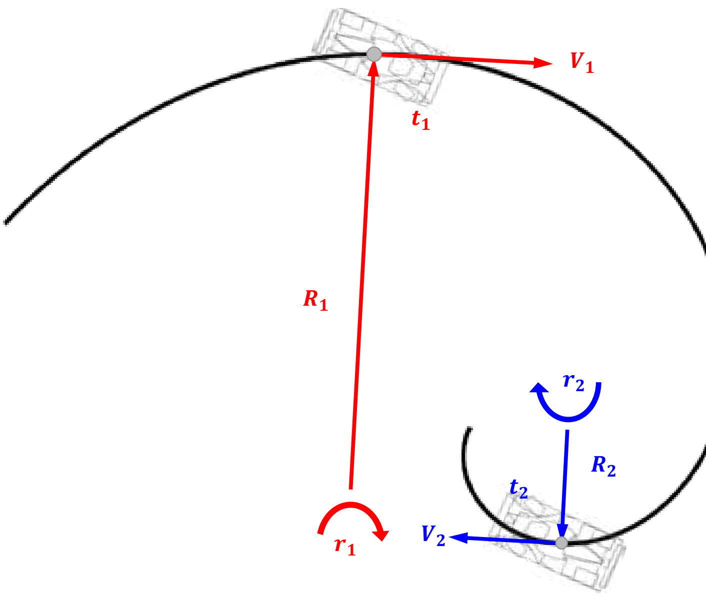

Figure 5: Same concept as Figure 4 but two skip pads do not have the same centre. Several iterative calculations can be made between points 1 and 2

skip pad but with a variation of the chassis slip angle. If in all three cases the lap time is the same (360 degrees in the same amount of time), the yaw velocity is different in the third case because of the variation of the chassis slip angle ß.

Figure 2 shows the difference between the math channel A/V (in red) and the gyro (in blue).

Let's now look at the yaw moment. The rotational perspective of the Newton second law **F = ma is N= Izz (dr/dt)** (5) where N is the yaw moment (in Nm), Izz is the yaw inertia (in kgm^2) and dr/dt is the derivative of the yaw velocity or the yaw acceleration.

Equation 5 could be written as **N = Izz [d(A/V) / dt + d²ß/dt²]** (6).

Theoretically, the yaw moment should be zero on the skid pad. A, V, R, ß and r are constant, or close to constant if we ignore the slight steering and throttle changes made by the driver. The car behaviour on skip pad is the closest situation of the steady state definition.

The steady state skip pad example is quite theoretical. No track is perfectly smooth, no driver input constant, wheels are never perfectly balanced and tyre grip (mainly temperature sensitive) is never constant. The reality is a car is practically in a transient state most of the time. So, two questions arise: how much is, or should, the value of the yaw moment be in transient and what are the parameters that influence the car yaw moment?

Yaw move

There are 12 causes for the yaw moment: four tyre lateral forces Fy, four tyre longitudinal forces Fx, and four tyre self-aligning torques Mz **(Figure 3)**. Looking at the car from the top, if we choose the car CG as a reference and we calculate the yaw moment around that point, distance a (from the front axle to the car CG) will be the leverage of the front tyre lateral force Fy (let's be careful to input the component of the front tyre lateral force that is perpendicular to the chassis longitudinal axis, not perpendicular to the front wheel), distance b (from the car CG to the rear axle) will be the leverage of the rear tyre lateral force Fy and each ½ track will be the leverage of each respective tyre longitudinal force Fx.

To answer the question on what should the yaw moment in transient be, let's imagine a car that is in steady state at a speed V_1 on a skip pad of a radius R_1, with a lateral acceleration Ay_1 and a yaw velocity r_1 **(Fig 4)**. We will now ask the driver to go as quickly as possible without *under* or *oversteer* to another skid pad that has a shorter radius R_2 on which he will reach another steady state with a speed V_2, a lateral acceleration Ay_2 and a yaw velocity r_2. Practically the driver must find the right combination of steering torque and brake pedal pressure to get the maximum deceleration and the yaw moment needed at any time.

Yaw moment

Going from a Speed V_1 to a Speed V_2 in a minimum of time Δt implies a longitudinal deceleration **Ax = (V_1 - V_2)/Δt.** Also, the car will go in this minimum amount of time Δt from a yaw velocity r_1 to a yaw velocity r_2 which implies a yaw acceleration **dr/dt = (r_1-r_2)/Δt**. Multiply this yaw acceleration by the yaw inertia and we get the yaw moment that is needed. We could imagine a similar transient behaviour in acceleration from skid pad two to skip pad one in acceleration.

We do not race on skid pad or skip pads, nevertheless the principle remains the same. If a driver follows a given trajectory (**Figure 5)** there will be changes in speed V, changes in radius R and therefore changes of yaw velocity, thus a need for a different yaw acceleration.

An understeering car has a deficit of yaw moment, an oversteering car has an excess of yaw moment.

The goal that racecar engineers and race drivers try to achieve is double; exploiting the tyres' potential forces and moments to get the best possible lateral, longitudinal or combined accelerations, and also to get the yaw moment they want, when they want it.

The reality is that a racecar is practically in transient behaviour most of the time

CONTACT
Claude Rouelle
Phone: + 1 303 752 1562
Enquiries: engineering@optimumg.com
Website: www.optimumg.com
OPTIMUM G